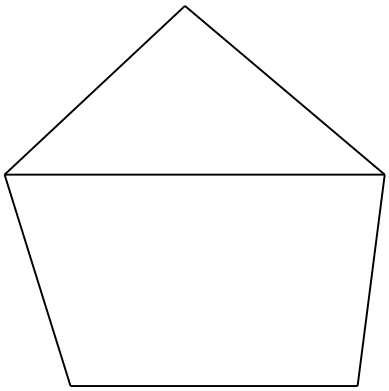
WISCONSIN MATHEMATICS, SCIENCE AND ENGINEERING TALENT SEARCH
PROBLEM SET V (2005-2006) **FEBRUARY 2006**

1. A club with seven members wants to form a number of three-person committees, but they require that no two committees should have more than one member in common. For example, if the members of the club are A, B, C, D, E, F and G, they could form these five committees: {A,B,G}, {A,C,F}, {A,D,E}, {B,C,E} and {D,F,G}. What is the maximum number of committees the club can form? Prove that your answer is correct.

2. Let us say that a diagonal of a pentagon is *good* if it is parallel to one of the sides of the pentagon. Show that if four of the five diagonals of a pentagon are good, then the fifth diagonal is also good.



3. Jake tells Jenny that he has three children, two of whom are twins, and that their ages are integers. He also tells her the sum of the ages of his children and the product of their ages. Jenny says that she does not have enough information to determine the ages, but one possibility is that the twins are a prime number of years old. If Jake's twins are teenagers and their age is not prime, find (with proof) the ages of his children.

4. Let A and B be points in the plane at distance 2 from each other. Let S be the set of points P such that $(PA)^2 + (PB)^2$ is at most 10. What is the area of S?

5. If x, y and z are positive real numbers, show that

$$\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} \geq \frac{3}{2}.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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