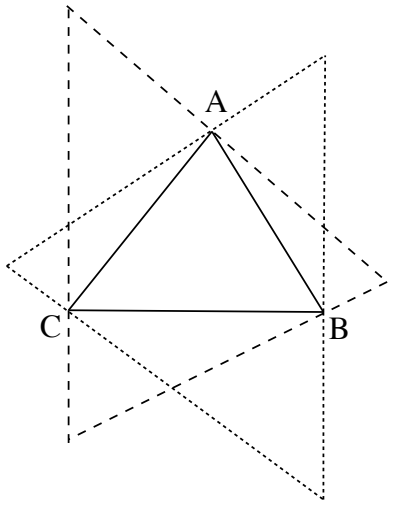
WISCONSIN MATHEMATICS, SCIENCE AND ENGINEERING TALENT SEARCH
PROBLEM SET IV (2005-2006) **JANUARY 2006**

1. Show that for each odd prime number p, there is exactly one positive integer n such that $n(n+p)$ is a perfect square.

2. Given $\triangle ABC$, we build two new triangles as follows. First, draw lines through A, B and C perpendicular to $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, respectively. These three lines form one of our new triangles. The other one is also formed by lines through A, B and C, but this time the lines are perpendicular to $\overline{CA}$, $\overline{AB}$ and $\overline{BC}$ respectively. Prove that the two new triangles are congruent.



3. Jake tells Jenny that he has three children, two of whom are twins, and that the ages of all three children are integers. Jake also tells Jenny the sum and the product of the ages of his children. Jenny then says that the age of the non-twin child must be either 9 or 25, but that there is not enough information to determine which of these is correct. Determine (with proof) the product of the ages of Jake's children. [This is a corrected version of the original problem.]

4. New Year's Problem. Prove that for every positive integer m the number $(\sqrt{2006}+\sqrt{2005})^{2m}$ differs from an integer by no more than $1/(4{\cdot}2005)^m$.

5. Let $S = \{2, 3, 22, 23, 32, 33, 222, \ldots\}$ be the set of all positive integers whose decimal digits are 2s and 3s only. Show that no three distinct members of S are in arithmetic progression. (Recall that three integers $a \le b \le c$ are said to be in *arithmetic progression* if $c - b = b - a$.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu
Please Fill In	**PROBLEM SET IV**
Name & Grade	
School & Town	
Home Address	
Town & Zip	
Email Address	

Deadline February 3, 2006	
Problem	**Score**
1	
2	
3	
4	
5	