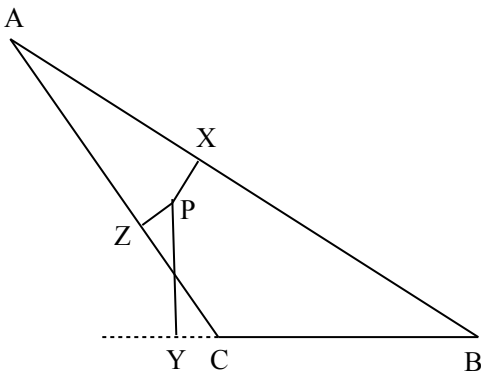
WISCONSIN MATHEMATICS, SCIENCE AND ENGINEERING TALENT SEARCH
PROBLEM SET III (2005-2006) **DECEMBER 2005**

1. Find all real solutions (if there are any) for

$$\frac{w^2+1}{xy} = \frac{x^2+1}{yz} = \frac{y^2+1}{zw} = \frac{z^2+1}{wx} = 2\,.$$

2. Let l and s be the lengths of the longest and shortest altitudes, respectively, of triangle ABC, and let P be a point in the interior of the triangle. Suppose perpendiculars $\overline{PX}$, $\overline{PY}$ and $\overline{PZ}$ are dropped from P to the sides of the triangle. (If the angles of the triangle are not acute, it may be necessary to extend the sides, as shown.) Prove that $s \le PX + PY + PZ \le l$, and that both inequalities are strict unless the triangle is equilateral.



3. Find all numbers c such that the equation $2x+3y = c$ has exactly 1000 solutions in positive integers.

4. Let $n \ge 0$ be an integer and consider numbers of the form $20\cdots04$, where there are n zeros between the initial 2 and final 4. Determine which of these numbers can be written as products of four positive integers in arithmetic progression. Recall that a, b, c and d are said to be in arithmetic progression if $b - a = c - b = d - c$.

5. Let S be a string of 2s and 3s such as, for example, 2322. We assign a value $v(S)$ to S by the following process. First, we compute the numbers a_i equal to the sum of the first i digits in the string, starting from the left. Thus in our example, $a_1 = 2$, $a_2 = 2+3 = 5$, $a_3 = 2+3+2 = 7$ and $a_4 = 2+3+2+2 = 9$. The value assigned to the string is the sum of the numbers a_i, so that in our example, $v(S) = 2+5+7+9 = 23$. Find all positive integers that are not values of strings of 2s and 3s.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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