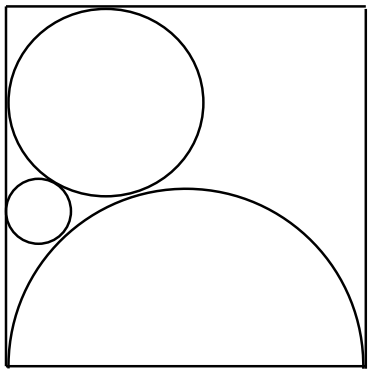
WISCONSIN MATHEMATICS, SCIENCE AND ENGINEERING TALENT SEARCH
PROBLEM SET II (2005-2006) **NOVEMBER 2005**

1. Find all real solutions (if there are any) for

$$\frac{w^2+1}{xy} = \frac{x^2+1}{yz} = \frac{y^2+1}{zw} = \frac{z^2+1}{wx} = 1\,.$$



2. In the previous problem set, we had a square with side length 2, where the lower side of the square was the diameter of a semicircle. A circle tangent to the semicircle and to two sides of the square was drawn, and we saw that its radius was $4-2\sqrt{3}$. Now we add a smaller circle, tangent to the semicircle, the first circle and one side of the square, as shown. Show that the radius of the small circle is exactly 1/3 the radius of the large circle.

3. When the UW football team scores, Bucky Badger does a number of pushups equal to UW's total score at that time. For instance, if Wisconsin scores 3 points, then 7 points and then 3 points again, Bucky does first 3, then $3+7=10$ and then $3+7+3=13$ pushups, for a total of $3+10+13=26$. If all of UW's scores are either 3 or 7 points and Bucky does 71 pushups during the game, how many points did UW score in that game?

4. The number of "words" of length n that use only the letters X and Y is exactly 2^n since there are two choices for each of the n positions. Let $F(n)$ be the number of these words that contain two consecutive Xs. For example, if $n=4$, the words with consecutive Xs are XXXX, XXXY, XXYX, XYXX, YXXX, XXYY, YXXY and YYXX, so $F(4)=8$. Find $F(10)$ and justify your answer.

5. Alan and Betty play a game by taking turns removing stones from a pile. The player who takes the last stone loses, and the rules require that at each turn, the number of stones removed must be either 1/3 or 1/4 of the pile, rounded up to the next integer. (For example, if eight stones remain, the number that can be removed is either $8/4=2$ or $8/3$ rounded up, which is 3.) If Alan goes first and the pile initially has 100 stones, show that Betty can win regardless of what Alan does.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions usually require a proof or justification.

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Deadline December 1, 2005	
Problem	**Score**
1	
2	
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