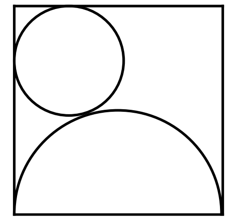
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2005-2006) — October 2005

1. Let S be some set of positive integers and let T be the set of all numbers of the form $x + y$, where x and y are in S, allowing $x = y$. Suppose that T is the complement of S in the set of positive integers. (In other words, T is exactly the set of positive integers that do not lie in S.) Show that the number $1,000,000$ is a member of T.

2. In the diagram, the length of each side of the square is 2 and the bottom side of the square is the diameter of the semicircle. The circle is tangent to the semicircle and to two sides of the square, as shown. Compute (with proof) the radius of the circle.



3. One season, each team in a sports league won at least five games against other teams in the league. Prove that some team lost at least five games that season.

4. Let a be an odd positive integer and suppose that the equation $x^2 - y^2 = a$ has exactly one solution in positive integers x and y. Prove that either a is a prime number or it is the square of a prime number.

5. Consider numbers $a_1 < a_2 < a_3 < a_4$ and $b_1 < b_2 < b_3 < b_4$. Suppose also that $a_2 < b_1$ and that $a_4 < b_4$. If $a_1 + a_2 + a_3 + a_4 \geq b_1 + b_2 + b_3 + b_4$, determine the ordering of these eight numbers. In other words, say which is the smallest, the second smallest, the third smallest, *etc.* (Of course, a proof is required here.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

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DEADLINE: November 1, 2005

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(Please Detach Above)

Last Name   First Name   Grade

School   Town

Home Address   Town   Zip Code

Email Address

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET I