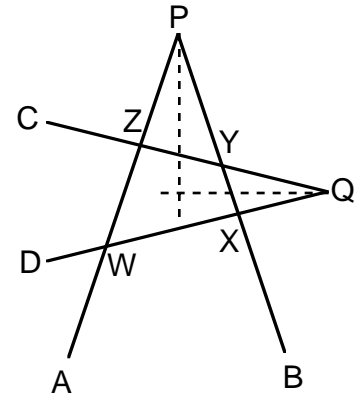


WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2004-2005)

FEBRUARY 2005

- Let $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$ be the set of integers, positive, negative and 0. A *subgroup* S of Z is a nonempty subset with the property that if x and y are members of S , then so is $x - y$. Find all subgroups of Z that contain the integer 3.
- The sides of $\angle APB$ and $\angle CQD$ meet at points W, X, Y and Z , as shown. If the bisectors of these angles are perpendicular, show that the four points W, X, Y and Z lie on a common circle.
- Let $N = 100\dots001$ be the integer having $n \geq 0$ zero digits sandwiched between the two ones. If N is a prime number, prove that $n + 1$ is a power of 2.
- Show that no sum of reciprocals of squares of distinct positive integers can ever be as large as 2.
- Prove that there are infinitely many pairs of integers x, y satisfying the equation $x^2 - 2y^2 = 1$.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
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 University of Wisconsin, Madison, WI 53706
 OR: talent@math.wisc.edu

DEADLINE:
 March 7,
 2005

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 (Please Detach Above)

Last Name	First Name	Grade
School	Town	
Home Address	Town	Zip Code
Email Address		

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V