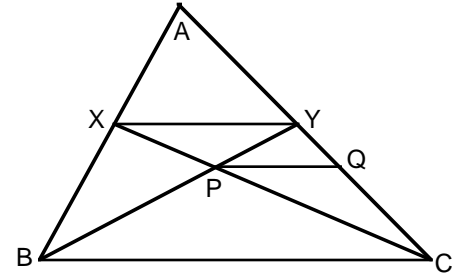


WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET II (2004-2005) **NOVEMBER 2004**

- Do there exist primes p and q such that the quadratic equation $px^2 - qx + p = 0$ has a rational (i.e. a ratio of integers) solution? If so, find all possibilities.
- In $\triangle ABC$, line \overline{XY} is drawn parallel to \overline{BC} , where X lies on side \overline{AB} and Y lies on side \overline{AC} , as shown. Let P be the point where \overline{BY} meets \overline{CX} , and suppose that the line through P parallel to \overline{BC} meets \overline{YC} at Q . If $CQ = 2$ and $QY = 1$, find length AC and prove that your answer is correct.
- Given a real number x , the *floor* of x , denoted $\lfloor x \rfloor$, is defined to be the largest integer n such that $n \leq x$. Similarly, the *ceiling* of x , denoted $\lceil x \rceil$, is the smallest integer m such that $x \leq m$. We also define $\text{av}(x)$ to be the average of $\lfloor x \rfloor$ and $\lceil x \rceil$. Prove that $\lfloor x + y \rfloor \leq \text{av}(x) + \text{av}(y) \leq \lceil x + y \rceil$ for all real numbers x and y .
- Recall that the *Fibonacci numbers* are the sequence $1, 1, 2, 3, 5, 8, 13, 21, \dots$, where after the initial two 1's, each number in the sequence is the sum of the previous two. Prove that there is no positive integer m such that the sum of every m consecutive Fibonacci numbers is odd. Also, determine all positive integers n such that the sum of every n consecutive Fibonacci numbers is even.
- Let A, B and C be subsets of a finite set S and assume that every element of S is in at least one of A, B or C . Prove that there is a subset X of the set $\{A, B, C\}$ such that at least $4/7$ of the elements of S are in an odd number of the members of X .



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
 Dept. of Mathematics, 480 Lincoln Drive
 University of Wisconsin, Madison, WI 53706
 OR: talent@math.wisc.edu

DEADLINE:
 December 1,
 2004

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 (Please Detach Above)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code
Email Address		

PROBLEM	SCORE
1	
2	
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PROBLEM SET II