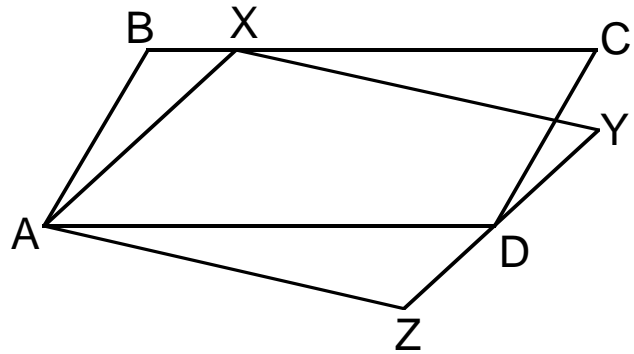


**WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET I (2004-2005)**

**OCTOBER 2004**

- Find all positive integers  $c$  such that it is possible to write  $c = \frac{a}{b} + \frac{b}{a}$  with positive integers  $a$  and  $b$ .
- In the figure,  $ABCD$  and  $AXYZ$  are parallelograms such that point  $X$  lies on side  $\overline{BC}$  and point  $D$  lies on side  $\overline{YZ}$ . Prove that the two parallelograms have equal areas.



- Find all prime numbers  $p$  such that  $p^2 = n^3 + 1$  for some integer  $n$ .
- Find all real numbers  $a$  with the property that the polynomial equation  $x^{10} + ax + 1 = 0$  has a real solution  $r$  such that  $1/r$  is also a solution.
- Several people started with \$300 each, and played a game with the following strange rules. *Each player pays \$10 to the house at the beginning of each round. During each round, one active player is declared the loser, and he distributes all of his money in equal amounts to the remaining players. The loser must then leave, but all of the other players go on to the next round. The game is over when only one player remains. At the end of the game, the surviving player was surprised to discover that he had exactly \$300, equaling his starting amount. How many players were there at the beginning?*

**You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.**

RETURN TO:

MATHEMATICS TALENT SEARCH  
 Dept. of Mathematics, 480 Lincoln Drive  
 University of Wisconsin, Madison, WI 53706  
 OR: talent@math.wisc.edu

DEADLINE:  
 November 1,  
 2004

(Please Detach Above)

Last Name	First Name	Grade
School	Town	
Home Address	Town	Zip Code
Email Address		

PROBLEM	SCORE
1	
2	
3	
4	
5	

**PROBLEM SET I**