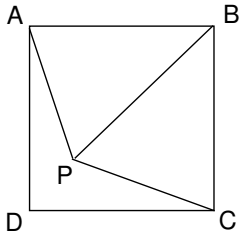
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2003-2004) **FEBRUARY 2004**

1. Ten people attend a party at which some of the guests shake hands with other guests and no two people shake hands with each other more than once. If there are a total of 26 handshakes, prove that there must be three people at the party each of whom has shaken hands with both of the others.

2. In the diagram, $ABCD$ is a square and P is a point inside it. Show that the distances PA, PB and PC satisfy the inequality $PA + PC \geq \sqrt{2} \cdot PB$. (Actually, it is irrelevant that P is *inside* the square. The inequality is valid for all points P in the plane.)



3. Suppose that p and q are prime numbers and that m is an integer. If $n = \frac{p}{q} + \frac{q}{p} - \frac{m^2}{pq}$ is a positive integer, find all possibilities for n.

4. Let f be a function such that $f(n)$ is an integer for each integer n. Prove that there is some integer n such that $f(f(n)) \neq n + 3$.

5. All of an 8×8 chessboard, with the exception of one square, is covered by 1×3 rectangular tiles. How many of the 64 squares can occur as the uncovered square?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

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DEADLINE: March 10, 2004

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(Please Detach Above)

Last Name	First Name	Grade

School		Town

Home Address	Town	Zip Code

Email Address

PROBLEM	SCORE
1	
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PROBLEM SET V