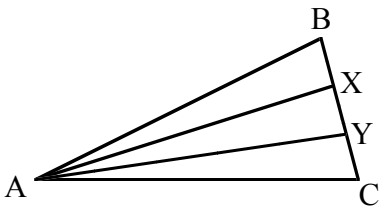
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2003-2004) **JANUARY 2004**

1. (New Year's Problem) Consider the following numbers, each of which is slightly larger than 5/3: $a_0 = 1.7$, $a_1 = 1.67$, $a_2 = 1.667$ and in general, $a_m = 1.66\cdots67$, where there are exactly m sixes in a_m. Find the integer of the form na_m that is closest to 2000, where n and m are integers. Prove that your answer is correct.

2. Let $\overline{AX}$ and $\overline{AY}$ be the trisectors of $\angle A$ in $\triangle ABC$, as shown. Prove that it is *not* true that $BX = XY = YC$.



3. Suppose that A and B are nonempty subsets of the set of positive integers, and assume that each positive integer is in one of these two sets, but that no integer lies in both. Suppose also that each of A and B is closed under triple sums. In other words, if x, y and z are any three (not necessarily different) members of A, then $x + y + z$ lies in A, and similarly for B. Find all possibilities for the sets A and B.

4. As in Problem 1 of the previous problem set, let a and b be positive numbers with $(1/a) + (1/b) = 1$, and let $[x]$ denote the largest integer that does not exceed the number x. If neither a nor b can be written as the ratio of two integers, show that each positive integer is either of the form $[ma]$ or $[mb]$ for some positive integer m.

5. Some of the people attending a party hate each other, but no one at the party hates more than three other guests. Prove that it is possible for all of the people at the party to assemble in two (large) rooms so that in each room, no individual hates more than one other person in that room. Assume that the hatred relation is symmetric, which means that if P hates Q, then also Q hates P.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

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DEADLINE: February 5, 2004

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(Please Detach Above)

Last Name | First Name | Grade

School | Town

Home Address | Town | Zip Code

Email Address

PROBLEM	SCORE
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PROBLEM SET IV