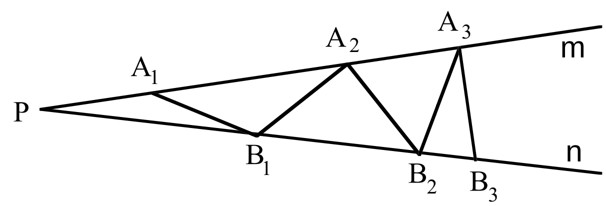
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2003-2004) **DECEMBER 2003**

1. If x is any real number, we write $[x]$ to denote the largest integer that does not exceed x. Suppose that a and b are positive numbers such that $(1/a) + (1/b) = 1$. If m and n are positive integers such that $[ma] = [nb]$, show that ma and nb are integers.

2. Lines m and n meet at point P as shown, and we perform the following construction. Point A_1 is chosen on line m so that the distance $PA_1 = 1$. Then a point B_1, different from P, is chosen on line n so that $A_1B_1 = 1$. Next, point A_2, different from A_1, is chosen on m so that $B_1A_2 = 1$, and similarly B_2 is chosen on n with $A_2B_2 = 1$. This process is continued for as long as possible, with each point farther from P than the previous one. Note that if $\angle P = 60°$, then $PA_1 = PB_1$. (a) If $PA_2 = PB_2$, compute $\angle P$. (b) If $PA_{10} = PB_{10}$, compute $\angle P$.



3. Suppose that $an^3 + bn^2 + cn + d$ is an integer for every integer $n \geq 1,000,000$. Prove that all of the numbers $6a$, $6b$, $6c$ and $6d$ are integers.

4. Find all numbers a such that the equation $\Big|x + |x| + a\Big| + \Big|x - |x| - a\Big| = 2$ has exactly three solutions.

5. Let x, y and z be positive. Prove that

$$(x^2 + y^2 + z^2)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \geq 3(x + y + z)\,.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

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DEADLINE: January 5, 2004

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(Please Detach Above)

Last Name First Name Grade

School Town

Home Address Town Zip Code

Email Address

PROBLEM	SCORE
1	
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PROBLEM SET III