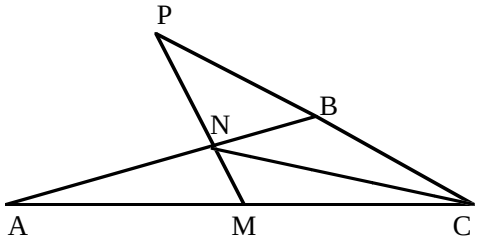
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2003-2004) **NOVEMBER 2003**

1. Find all positive integers n such that the number $n^4 + n^3 - 8$ is a perfect square. Prove that you have found all possibilities.

2. In the figure, point M lies on side $\overline{AC}$ of $\triangle ABC$ and point N lies on side $\overline{AB}$. These points are chosen so that the three small triangles $\triangle AMN$, $\triangle CMN$ and $\triangle CBN$ all have equal area. If line segments $\overline{MN}$ and $\overline{BC}$ are extended to meet at point P, show that B is the midpoint of $\overline{PC}$.



3. In the previous problem set, we were introduced to the strange spelling rules on the planet AZAAZ. In their language, the letter A can be replaced by ZAZ and the combination ZAZ can be replaced by A, and these transformations can be made as many times as desired. We saw that AAAZZ is a valid spelling for the name of the planet, but that ZAZAZ is not. Decide if AZAZA is a valid spelling for the name of the planet and prove that your answer is correct.

4. If x and y are any two integers, let $x \,\square\, y$ denote an integer determined by x and y. Assume that the binary operation $\square$ satisfies the rules: $x \,\square\, (y + z) = (x \,\square\, y) - z$, $(y + z) \,\square\, x = (y \,\square\, x) + 2z$, and $1 \,\square\, 1 = 1$. Compute $25 \,\square\, 10$.

5. Find all positive real numbers x, y and z that satisfy the following three equations: $x + \frac{4}{xy} = 3$, $y + \frac{4}{yz} = 3$, and $z + \frac{4}{zx} = 3$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
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DEADLINE: December 2, 2003

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(Please Detach Above)

Last Name	First Name	Grade

School		Town

Home Address	Town	Zip Code

Email Address

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET II