

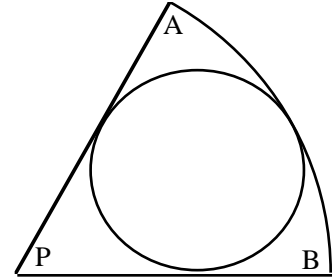
**WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET I (2003-2004)**

**OCTOBER 2003**

1. Suppose that  $n$  is a positive integer with the property that there are exactly six different positive integers  $m$  such that  $n/m$  is an integer. If one of these six numbers is  $m = 27$ , find all possibilities for  $n$ .

2. In the figure,  $APB$  is a sector of a circle centered at  $P$ . This means that  $\overline{PA}$  and  $\overline{PB}$  are radii and that  $AB$  is an arc of the circle. A smaller circle is inscribed in the sector, as shown, so that it is tangent to  $\overline{PA}$ ,  $\overline{PB}$  and arc  $AB$ . Given that  $\angle APB = 60^\circ$ , find the fraction of the area of the sector that is covered by the inscribed circle.



3. Suppose that  $a^2 + b^2 + c^2$  is a multiple of 16, where  $a$ ,  $b$  and  $c$  are integers. Show that  $a^3 + b^3 + c^3$  is a multiple of 64.

4. Every word in the language of the planet AZAAZ can be spelled using just the letters A and Z, and most words can be spelled in many different ways. This is because the letter A can always be replaced by ZAZ and the combination ZAZ can always be replaced by A. These replacements can be done repeatedly, but no others are allowed. For example, the capital city of AZAAZ is ZAZA, and this name can also be spelled AA or AZAZ or ZAZAZ, and there are many other possible spellings. Prove that AAAZZ is a correct spelling for the name of the planet, but that ZAZAZ is not.

5. Decide whether or not the number  $a = \sqrt[3]{99 - 70\sqrt{2}} + \sqrt{8}$  is an integer and prove that your answer is correct.

**You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.**

RETURN TO:

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DEADLINE:  
 November 1,  
 2003

(Please Detach Above)

Last Name	First Name	Grade
School	Town	
Home Address	Town	Zip Code
Email Address		

PROBLEM	SCORE
1	
2	
3	
4	
5	

**PROBLEM SET I**