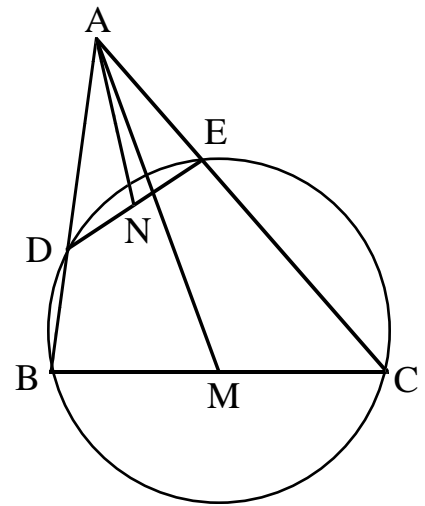


WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2002-2003)

FEBRUARY 2003

1. An integer n is a sum of two squares if there are integers x, y (possibly 0) with $n = x^2 + y^2$. If each of the integers a and b is a sum of two squares, show that their product ab is also a sum of two squares. Prove that the number $5^{64} - 3^{64}$ is a sum of two squares.
2. A circle is drawn through vertices B and C of $\triangle ABC$, and this circle meets sides \overline{AB} and \overline{AC} at points D and E , as shown. If the midpoint of \overline{BC} is M and the midpoint of \overline{DE} is N , show that $\angle DAN = \angle CAM$.
3. The cubic polynomial $p(x) = x^3 - 4x^2 + 2$ has three distinct real roots, say a, b and c . Find $a^4 + b^4 + c^4$.
4. Suppose that each positive integer is colored either red or blue. Show that there exist three positive integers $x < y < z$ having the same color and such that $z - y = y - x$.
5. Alice and Bob play a game by taking turns removing some stones from a pile. The rules require that the number of stones removed at each turn must be either 1, 2, 3 or 4, and the winner of the game is the person who takes the last stone. If we start with 100 stones and Alice goes first, prove that Bob can win, no matter what Alice does.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
 Dept. of Mathematics, 480 Lincoln Drive
 University of Wisconsin, Madison, WI 53706

DEADLINE
 March 14
 2003

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 (Please Detach)

Last Name	First Name	Grade
School		Town
Home Address		Town
		Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V