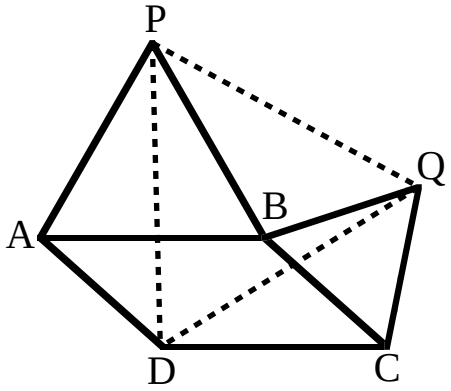
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2002-2003) **JANUARY 2003**

1. Find the smallest positive integer m such that m is not a square and yet in the decimal expansion of $\sqrt{m}$, the decimal point is followed by at least four consecutive zeros.

2. Equilaterial triangles $\triangle PAB$ and $\triangle QBC$ are constructed on two sides of the parallelogram $ABCD$, as shown. Prove that $\triangle DPQ$ is also equilaterial.



3. (New Year's Problem). Find the number of nonnegative integers n such that $2003 + n$ is a multiple of $n + 1$.

4. Let F_n be the n th Fibonacci number, so that $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$ and in general, $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. Let S be a set consisting of finitely many different Fibonacci numbers F_s with $s \geq 2$. Assume that S has more than one member and that the sum of all of the members of S is a Fibonacci number. Show that S must contain both F_{k-1} and F_k for some integer k.

5. Let f be a function which assigns to each positive integer n a positive integer $f(n)$. We suppose that $f(ab) = f(a)f(b) - f(a) - f(b) + 2$ for all positive integers a, b and that $f(c!) = c! + 1$ for all $c \geq 10^{10}$. Show that $f(n) = n + 1$ for all n.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
February 10
2003

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET IV