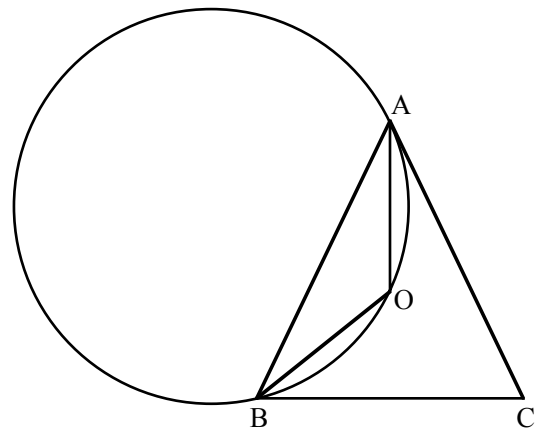


**WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET III (2002-2003)**

**DECEMBER 2002**

1. Find the smallest possible value for the expression  $(x + y + z)(xy + xz + yz)/(xyz)$  as  $x, y$  and  $z$  run over all positive real numbers.
2. Recall that it is always possible to draw a circle through the three vertices of a triangle. This is the *circumcircle* of the triangle and its center is called the *circumcenter* of the triangle. Suppose that point  $O$  is the circumcenter of the isosceles triangle  $\triangle ABC$ , where  $AB = AC$ . Show that line  $\overline{AC}$  is tangent to the circumcircle of  $\triangle AOB$ .



3. Decide whether or not the number  $\sqrt[3]{26 - 15\sqrt{3}} + \sqrt[3]{26 + 15\sqrt{3}}$  is an integer. Prove that your answer is correct.
4. If  $n$  is a positive integer, let  $d(n)$  denote the number of positive divisors of  $n$ . For example,  $d(12) = 6$  since the six positive divisors of 12 are 1, 2, 3, 4, 6 and 12. Prove that  $d(n) < 2\sqrt{n}$  for all positive integers  $n$  and find all positive integers  $n$  such that  $d(n) \geq 2\sqrt{n} - 1$ .
5. Alice and Bob play a game by taking turns removing some stones from a pile. The rules require that the number of stones removed at each turn must be a divisor of the number of stones in the pile at the start of that turn, and no player is ever allowed to take all of the stones. The winner of the game is the last person who takes a stone. If we start with 100 stones and Alice goes first, prove that Alice can win, no matter what Bob does.

**You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.**

RETURN TO:

MATHEMATICS TALENT SEARCH  
 Dept. of Mathematics, 480 Lincoln Drive  
 University of Wisconsin, Madison, WI 53706

DEADLINE  
 January 6  
 2003

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 (Please Detach)

Last Name	First Name	Grade
School	Town	
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

**PROBLEM SET III**