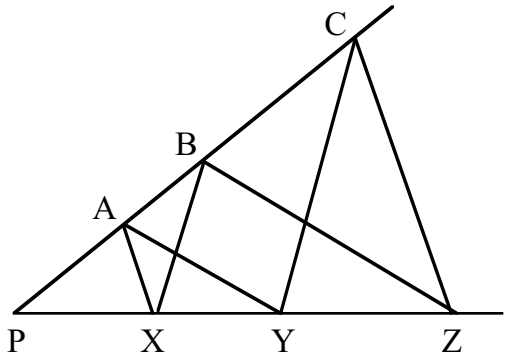
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2002-2003) **NOVEMBER 2002**

1. For any positive integer n, let $S(n)$ denote the sum of its digits. Show that the equation $n + S(n) = 1{,}000{,}000$ has no solution. Then solve the equation $n + S(n) = 1{,}000{,}000{,}000$.

2. Points P, A, B and C are collinear, as are the points P, X, Y and Z. If $\overline{AY}$ and $\overline{BZ}$ are parallel, and if $\overline{BX}$ and $\overline{CY}$ are parallel, show that the line segments $\overline{AX}$ and $\overline{CZ}$ are also parallel.



3. Find all real numbers x that satisfy $|x| - |x-1| + 2|x-2| = 3$.

4. Let $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$ and in general, $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. (This is the famous *Fibonacci sequence*.) Show that F_{2n}/F_n is always an integer.

5. Let a, b and c be fixed nonnegative integers, and assume that $n^3 + an^2 + bn + c$ is a perfect cube for infinitely many nonnegative integers n. Show that $n^3 + an^2 + bn + c$ is a perfect cube for all integers n.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
December 2
2002

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET II