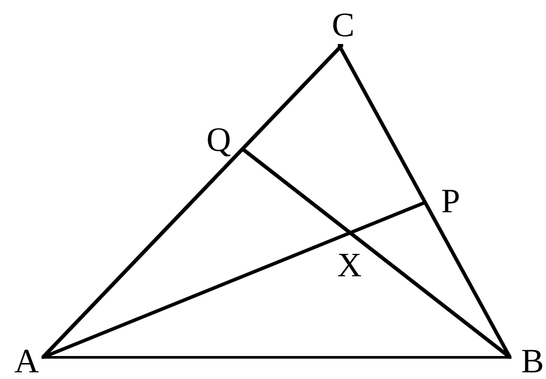
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2002-2003) **OCTOBER 2002**

1. In how many different ways can \$100.00 be made from 5-cent, 10-cent and 25-cent coins if it is required that exactly 1000 coins are used?



2. In triangle ABC, let P be the point on side $\overline{CB}$ satisfying $CP = (1/2)CB$ and let Q be the point on side CA with $CQ = (1/3)CA$. Draw the lines $\overline{AP}$ and $\overline{BQ}$, as indicated and suppose these lines meet at the point X. Find the ratio of the area of $\triangle ABX$ to the area of $\triangle ABC$.

3. Let x and y be nonzero real numbers satisfying the equation $x + 6/x = 2y + 3/y$. If $x/y \neq 2$, find the product xy.

4. Let $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, and in general, $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. (This is the famous *Fibonacci sequence*.) Show that $F_n/F_{n-1} < 1.7$ for all $n \geq 4$.

5. Find all positive integers n such that $n^2 + 25\,n + 19$ is a perfect square.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE November 1 2002

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(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET I