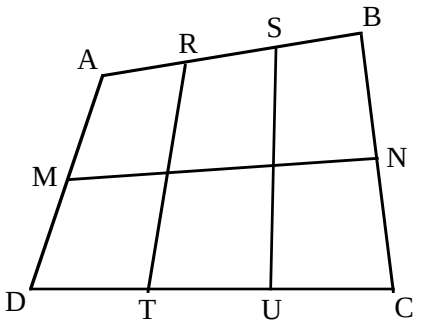
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2000-2001) **FEBRUARY 2001**

1. Let $a_1 = 14$, $a_2 = 144$ and in general, let a_n be the number $1444\cdots4$, where there are n fours. Find all positive integers n such that a_n is a perfect square.

2. Let $ABCD$ be a quadrilateral. Suppose that points M, N, R, S, T and U are selected on the sides of the quadrilateral as shown, where M and N are the midpoints of sides $\overline{AD}$ and $\overline{BC}$, respectively; points R and S trisect side $\overline{AB}$ and points T and U trisect side $\overline{DC}$. Show that line $\overline{MN}$ bisects line segments $\overline{RT}$ and $\overline{SU}$.



3. For each integer n there is defined a certain integer n^*, depending on n. Suppose that $1^* = 1$ and that $a^*b^* = (a+b)^* + (a-b)^*$ for all integers a and b. Compute 100^*.

4. Let X be a set of positive integers with the property that for every nonempty finite subset Y of X, the average of all the numbers in Y is an integer. Show that there exists such a set X containing 1000 (different) numbers, but that it is impossible for X to be infinitely large.

5. Let a, b and c be positive numbers. Prove that

$$2(a^8 + b^8) \geq (a^3 + b^3)(a^5 + b^5)\,, \quad \text{and}$$
$$3(a^8 + b^8 + c^8) \geq (a^3 + b^3 + c^3)(a^5 + b^5 + c^5)\,.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
March 14
2001

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(Please Detach)

Last Name First Name Grade

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PROBLEM	SCORE
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PROBLEM SET V