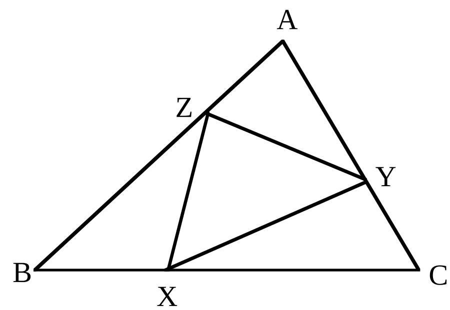
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2000-2001) **JANUARY 2001**

1. Suppose $\frac{a}{b} > \frac{x}{y} > \frac{c}{d}$ where a, b, c, d, x and y are non-negative integers. If $ad - bc = 1$, show that $x \geq a + c$ and $y \geq b + d$.

2. In $\triangle ABC$, point X lies on side $\overline{BC}$, one third of the way from B to C. Similarly, Y is chosen on $\overline{CA}$, one third of the way from C to A, and Z lies on $\overline{AB}$, one third of the way from A to B. If the area of $\triangle ABC$ is 1 unit, find the area of $\triangle XYZ$.



3. Let $\square$ be a binary operation defined on the set of nonnegative integers. (This means that if x and y are any two nonnegative integers, then $x \square y$ is a nonnegative integer determined by x and y.) Now suppose that (a) $(x+1) \square 0 = (0 \square x) + 1$, (b) $0 \square (y+1) = (y \square 0) + 1$, and (c) $(x+1) \square (y+1) = (x \square y) + 1$ are satisfied for all nonnegative integers x and y. If $1100 \square 450 = 2000$, find $1723 \square 3421$ and prove that your answer is correct.

4. Sixteen numbers are put into the boxes of a four-by-four array so as to form a magic square. This means that the four row sums, the four column sums and the two diagonal sums are each equal to the same number s. Show that s is also the sum of the four numbers in the corners of the array. (Do not assume that the sixteen numbers are the integers 1, 2, 3, . . . , 16.)

5. Consider polynomial equations of the form $x^3 + ax^2 + bx + 6 = 0$, where a and b are integers. Suppose that one of these equations has both r and $-r$ as roots, where r is a nonnegative real number. Find all possibilities for r.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	February 12
	University of Wisconsin, Madison, WI 53706	2001

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(Please Detach)

Last Name First Name Grade

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PROBLEM	SCORE
1	
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PROBLEM SET IV