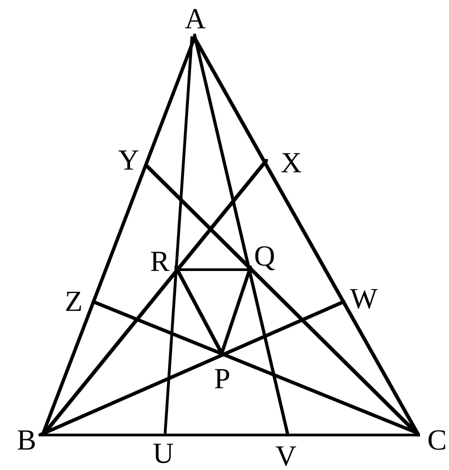
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2000-2001) **DECEMBER 2000**

1. Let m be a positive integer and suppose that $4m+1 = u^2+v^2$, where u and v are integers. Show that there exist integers a and b such that $m = \frac{a^2+a}{2} + \frac{b^2+b}{2}$.



2. In $\triangle ABC$, the points U and V trisect side $\overline{BC}$, points W and X trisect side $\overline{AC}$, and points Y and Z trisect side $\overline{AB}$. Points P, Q and R, as shown, are intersection points of lines joining vertices A, B and C to the trisection points on the opposite sides. Prove that the sides of $\triangle PQR$ are parallel to the sides of $\triangle ABC$.

3. Let a, b and c be positive numbers. Show that

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq a+b+c.$$

4. (New Year's Problem) How many pairs of positive integers a and b are there such that $a < b$ and $\frac{1}{a} + \frac{1}{b} = \frac{1}{2001}$?

5. Let S be a set of 100 positive integers, each less than 200. Show that there exists a nonempty subset T of S such that the product of all of the numbers in T is a perfect square.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
January 8
2001

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET III