1. Let $m$ be a positive integer and suppose that $4m + 1 = u^2 + v^2$, where $u$ and $v$ are integers. Show that there exist integers $a$ and $b$ such that $m = \frac{a^2 + a + b^2 + b}{2}$.

2. In $\triangle ABC$, the points $U$ and $V$ trisect side $BC$, points $W$ and $X$ trisect side $AC$, and points $Y$ and $Z$ trisect side $AB$. Points $P$, $Q$ and $R$, as shown, are intersection points of lines joining vertices $A$, $B$ and $C$ to the trisection points on the opposite sides. Prove that the sides of $\triangle PQR$ are parallel to the sides of $\triangle ABC$.

3. Let $a$, $b$ and $c$ be positive numbers. Show that

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq a + b + c.$$ 

4. (New Year’s Problem) How many pairs of positive integers $a$ and $b$ are there such that $a < b$ and $\frac{1}{a} + \frac{1}{b} = \frac{1}{2001}$?

5. Let $S$ be a set of 100 positive integers, each less than 200. Show that there exists a nonempty subset $T$ of $S$ such that the product of all of the numbers in $T$ is a perfect square.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

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DEADLINE: January 8, 2001

(Please Detach)