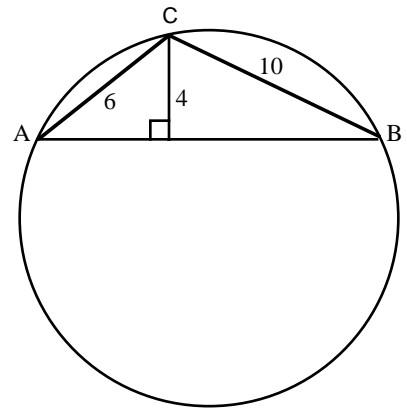


WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2000-2001)

NOVEMBER 2000

- Two race cars are traveling around a track at a constant speed, each of them taking one hour to complete one lap. One car starts at midnight and the other starts at 5:20 AM that morning. At what times after the second car starts is it true that the number of laps completed by the first car is exactly double the number of laps completed by the second car? At what times is it triple the number?



- In the circle, chord \overline{AB} is drawn and a point C on the circle is selected so that the perpendicular distance to the chord is 4 units, as indicated. If $CA = 6$ and $CB = 10$, find the diameter of the circle.

- Exactly how many numbers n in the range $1 \leq n \leq 1000$ can be written as $n = r^2 - s^2$ for positive integers r and s .

- Show that

$$\sqrt[3]{w^3 + x^3 + y^3 + z^3} \leq \sqrt{w^2 + x^2 + y^2 + z^2}$$

for all choices of real numbers w, x, y and z .

- Let us say that a positive integer n is *obtainable* if there exist integers x and y such that $n = 2x^2 + 3y^2$. If n is obtainable, prove that $7n$ is also obtainable.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
 Dept. of Mathematics, 480 Lincoln Drive
 University of Wisconsin, Madison, WI 53706

DEADLINE
 December 1
 2000

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET II