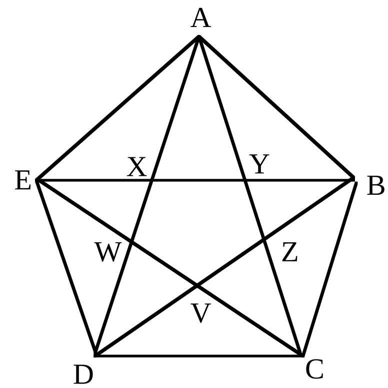
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2000-2001) **OCTOBER 2000**

1. Find all positive integers n such that $n^4 + n^3 + 1$ is a perfect square.



2. Let us say that a pentagon $ABCDE$ is *special* if each diagonal is parallel to its opposite side. In other word, the pentagon in the figure is special if and only if $EB \| DC$, $AC \| ED$, $BD \| AE$, $CE \| BA$ and $DA \| CB$. If $ABCDE$ is special, prove that the areas of the five triangles $\triangle AXY$, $\triangle BYZ$, $\triangle CZV$, $\triangle DVW$ and $\triangle EWX$ forming the points of the star are equal, and that $VWXYZ$ is a special pentagon.

3. Let A be the sum of ten positive real numbers and let B be the sum of the reciprocals of these ten numbers. What is the smallest possible value for AB?

4. A magic money machine does the following. If you put in a penny, out comes a dime; if you put in a dime, out come a penny and a quarter; and if you put in a quarter, out come two dimes. Starting with one dime, I play with the machine for a while. When I count my money, I find that I have exactly 100 pennies, along with other coins. What is the smallest total amount of money I could possibly have?

5. Find all pairs of nonnegative integers x and y such that $y^2(x + 1) = 1576 + x^2$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE November 1 2000

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET I