1. Find all positive integers \( n \) such that \( n^4 + n^3 + 1 \) is a perfect square.

2. Let us say that a pentagon \( ABCDE \) is *special* if each diagonal is parallel to its opposite side. In other word, the pentagon in the figure is special if and only if \( EB \parallel DC, AC \parallel ED, BD \parallel AE, CE \parallel BA \) and \( DA \parallel CB \). If \( ABCDE \) is special, prove that the areas of the five triangles \( \triangle AXY, \triangle BYZ, \triangle CZV, \triangle DVW \) and \( \triangle EWX \) forming the points of the star are equal, and that \( VWXYZ \) is a special pentagon.

3. Let \( A \) be the sum of ten positive real numbers and let \( B \) be the sum of the reciprocals of these ten numbers. What is the smallest possible value for \( AB \)?

4. A magic money machine does the following. If you put in a penny, out comes a dime; if you put in a dime, out come a penny and a quarter; and if you put in a quarter, out come two dimes. Starting with one dime, I play with the machine for a while. When I count my money, I find that I have exactly 100 pennies, along with other coins. What is the smallest total amount of money I could possibly have?

5. Find all pairs of nonnegative integers \( x \) and \( y \) such that \( y^2(x + 1) = 1576 + x^2 \).